

Mathematical Simulation of Membrane Protein Clustering for Efficient Signal Transduction

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Abstract—Initiation and propagation of cell signaling depends on productive interactions between signaling proteins at the plasma membrane. These diffusion-limited interactions can be influenced by features of the membrane that introduce barriers, such as cytoskeletal corrals, or microdomains that transiently confine both transmembrane receptors and membrane-tethered peripheral proteins. Membrane topographical features can lead to clustering of receptors and other membrane components, even under very dynamic conditions. This review considers the experimental and mathematical evidence that protein clustering impacts cell signaling in complex ways. Simulation approaches used to consider these stochastic processes are discussed.

Keywords—Clustering, Spatial stochastic simulations, Cell signaling.

INTRODUCTION

Cell signaling, used for both intracellular and intercellular communication, is essential for the healthy physiological functioning of multi-cellular organisms. Ligand binding to a transmembrane receptor triggers an intracellular signaling cascade that results in altered cell behavior. The proper integration of different environmental signals is critically important to many biological processes, including cell survival, differentiation, proliferation, and migration.^{10,39,42,49,85,89} Aberrations in signal transduction have been implicated in numerous pathologies, from allergy and asthma to many different

cancers.^{10,15,29,37,39,49,75,81,89} Signal transduction pathways have therefore been studied extensively, and many drugs developed to target them.^{10,22,29,49,75,81}

Knowledge of the structure of the plasma membrane and of signaling processes continues to improve, due to advances in experimental techniques and imaging technologies.^{46,84,87} There is considerable evidence for the concept that the cell membrane is compartmentalized into microdomains, such as protein islands⁸⁸ and lipid rafts.⁵⁶ Receptor clustering in small or large aggregates (illustrated schematically in Fig. 1) at discrete locations has been noted in many cell types,^{1,6,31,39,66,73,89} prompting intense interest in roles for membrane microdomains in signal propagation and preliminary mathematical studies to understand both formation of clusters and their role in cell signaling.^{8,17,18,21,35,36,48,63,76,77} There is general agreement that the composition of these microdomains is quite heterogeneous and, further, that their stability is influenced by the dynamic interactions of the cortical cytoskeleton with membrane proteins and lipids. The cytoskeleton also limits diffusion of membrane constituents by forming “picket fences” and “corrals.”^{43,72} The role of these membrane features in promoting or limiting protein–protein interactions remains controversial, since there is strong potential to both enhance and inhibit signaling.^{3,17,55,58} To help resolve these issues, several groups are developing spatially realistic mathematical simulations of receptor motion, aggregation/clustering, and activation in the cell membrane.

It is important to note that parameters for these mathematical models rely on powerful new experimental techniques. High resolution microscopy techniques, such as transmission electron microscopy

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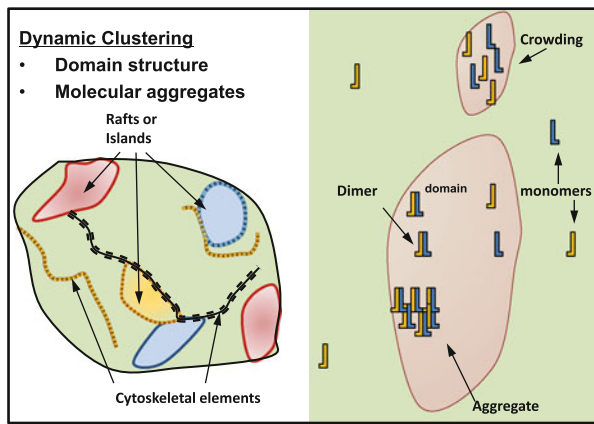


FIGURE 1. Schematic representation of microdomains and receptor clustering. Left: Cartoon representation of features that can subcompartmentalize the plasma membrane, including rafts or islands, and the cortical cytoskeletal network. These features are highly dynamic, permitting rapid exchange by diffusion. Right: Representation of the distribution of receptors (yellow, blue symbols) in and out of domains (pink shapes) formed by these features. Arrows point to various states, including monomers, dimers, and aggregates. Receptors that are transiently trapped in domains are locally crowded (arrow, top right) and appear as clusters by imaging technologies. This molecular crowding can be more pronounced upon ligand stimulation, due in part to formation of dimers and larger aggregates with decreased diffusive mobility. This review considers the experimental and computational evidence that molecular crowding influences receptor dimerization/aggregation and recruitment of signaling proteins.

(TEM) and photoactivation light microscopy (PALM), have been applied to map the spatial distribution of signaling molecules in fixed cells.^{47,88} These snapshot images of protein distributions can be supplemented with powerful new live cell imaging approaches, including fluorescence resonance energy transfer (FRET), fluorescence lifetime correlation spectroscopy (FLCS) and single particle tracking (SPT) experiments.⁴⁶ These techniques can generate key information regarding the kinetics of protein–protein interactions, including rates of dimerization, size of receptor aggregates, and changes in diffusion properties.⁵⁰ These rich data sets support the development of more accurate and detailed mathematical models, that in turn improve understanding of biological results.

Key Concepts and Definitions Relevant to the Consideration of Protein Clustering in the Plasma Membrane

In this brief review, we focus attention on the mathematical simulation of protein clustering in the plasma membrane, an initial step in many signaling pathways. The protein species considered may be a *surface receptor*, that is triggered by binding to an extracellular ligand, or could be an *intracellular*

signaling partner, such as an adaptor protein or enzyme that propagates signaling through the cell interior. We define *clustering* as the non-random spatial distribution of a membrane species, which can be observed and experimentally validated through modern technologies. “Snap-shot” images of membrane proteins often capture some level of clustering even before the onset of ligand binding to receptors or active signaling.⁸⁹ We hypothesize that these basal levels of clustering arise from brief, non-productive interactions among proteins as they encounter one another while diffusing in the plasma membrane or when proteins are transiently co-confined in a raft, island or corral (Fig. 1). Thus clustering in this sense is not synonymous with *oligomerization*, which reflects the direct and measurable interaction between membrane components. It is important to point out that stable oligomers cannot be distinguished from unstable clusters in imaging techniques using fixed cells, such as TEM and PALM. However, new imaging protocols can now accurately measure the dynamics of protein–protein interactions at the molecular scale.⁴⁶ A recent example from our Center is the simultaneous SPT of pairs of EGFR molecules, each labeled by virtue of binding to EGF conjugated to different colors of quantum dot probes; only when two EGF-QD-bound receptors were both coincident and exhibited correlated motion, could they pass the stringent criteria for oligomerization.⁵⁰ The concept of clustering becomes particularly important as we consider the data suggesting that the spatial proximity of proteins can *promote* protein–protein interactions, including oligomerization, by increasing the likelihood of productive collisions.

Choosing the Right Modeling Approach

Mathematical models constructed to date to study signal transduction pathways are of varied complexity. They can be classified conveniently as deterministic methods, in which inherent temporal and spatial fluctuations in diffusion and reaction rates are ignored, and stochastic methods, which attempt to capture these fluctuations (Fig. 2). The simplest modeling approach is to assume that the system of interest is well mixed, without any spatial concentration gradients, and describe the reactions by a system of ordinary differential equations (ODEs). The utility of ODE modeling is enhanced by systematic sensitivity analysis, which examines automatically changes in model behavior due to parameter variation.^{60,61} Such a deterministic, well-mixed approach continues to be widely used,⁷⁷ and has produced useful results.^{7,61} However, these approaches do not take into account either spatial inhomogeneities or stochastic fluctuations, which can be significant when the number of

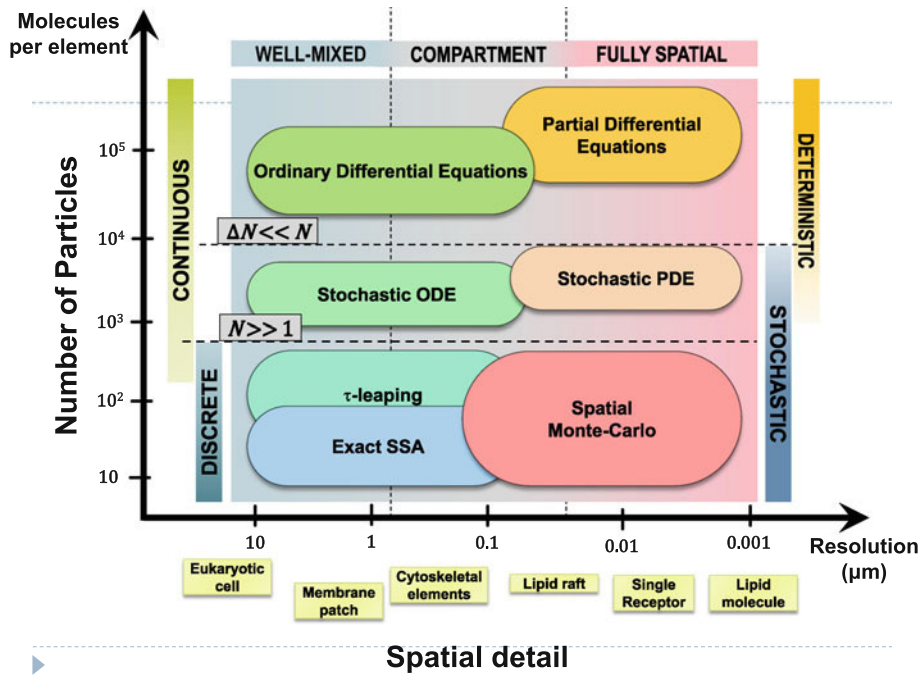


FIGURE 2. Classes of mathematical models for molecular processes in cells and the scales at which they are applicable to signaling processes. A possible quantitative guide is the size of the largest element that can be treated as spatially homogeneous (horizontal axis) and the typical number of molecules of one species in the element (vertical axis). The suggested spatial resolution is determined by the size of the biological elements of interest and current computational capabilities. Spatially resolved models are resource-intensive, and are therefore generally applied to small subsystems. Cell-level models of large signaling networks are typically well mixed; spatial Monte Carlo studies rarely scale beyond a few hundred nanometers. A promising approach for multi-scale applications is a combination of compartment-based models at the large scales and fully spatial simulations focused on a few important processes within small structural elements of the membrane. Temporal fluctuations arise largely from the discrete and stochastic nature of the underlying molecular processes. The relative magnitude of temporal fluctuations (ΔN) decreases as the number of particles increases. The discrete nature of the particle number can thus be ignored when N is significantly greater than 1. That is, deviations from the expected average behavior can be neglected when the expected magnitude of the fluctuations is small compared to N .

154 molecules in the region of interest is small. At a slightly
 155 higher level of complexity, some spatial description is
 156 provided by dividing the region of interest into separate
 157 well-mixed compartments. Additional ODEs are
 158 needed to describe inter-compartmental species trans-
 159 location reactions, thus mimicking spatial movement.

160 These well-mixed, ODE-based continuum pathway
 161 models⁴¹ were expanded to include spatial inhomoge-
 162 neity^{9,71} by solving partial differential equations
 163 (PDEs) that include molecular diffusion effects. Sto-
 164 chastic methods that assume spatially well mixed sys-
 165 tems have also been developed to account for temporal
 166 fluctuations.^{27,45} Stochastic PDEs include both spatial
 167 information and temporal fluctuations. The most
 168 detailed, and thus most complex, models are fully
 169 spatial, stochastic methods that track the movement of
 170 individual molecules.^{4,11,17,18,30,35,36,63,78} However, the
 171 computational burden increases rapidly with increas-
 172 ing complexity of the modeling approach. Figure 2
 173 summarizes the various modeling approaches and their
 174 range of applicability.

175 Mathematical simulation of events in the plasma
 176 membrane faces unique challenges. Membrane

177 proteins are constantly undergoing random motion in
 178 the plane of the membrane, where the diffusion rate is
 179 influenced by the environment, such as hindrance by
 180 microdomains, and thus varies both spatially and
 181 temporally. Optimally, the spatial location of every
 182 protein needs to be predicted, in order to capture
 183 clustering imposed by membrane topography and to
 184 predict the outcomes of both transient and prolonged
 185 protein-protein binding events. Fully spatial, stochas-
 186 tic methods offer capabilities that can capture accu-
 187 rately the dynamics of these events, but can be
 188 associated with prohibitively high computation cost.
 189 Novel hybrid approaches show promise for solving
 190 some of these computational challenges.

191 Finally, this section would not be complete without
 192 introducing the unique power of rules-based
 193 approaches.^{20,33} Here, molecular interactions in sig-
 194 naling networks are treated as systems of encoded
 195 rules. Molecules are represented as structural objects
 196 that have modular domains and associated states rep-
 197 resenting conformations or covalent modifications of
 198 these domains. Importantly the input files and model
 199 specification blocks are compatible with multiple types

200 of computational approaches, including coupled
201 ODEs that result in deterministic solutions of reaction
202 kinetics as well as stochastic methods.

203 APPLICATIONS IN SPECIFIC SIGNALING 204 PATHWAYS

205 Sections below briefly summarize mathematical
206 models that have been developed to study signal
207 transduction pathways, with emphasis on methods
208 developed by our group and others to capture the
209 influence of clustering and other spatial aspects. We
210 focus on three representative signal transduction
211 pathways (EGFR, Ras/MAPK, and GPCR) where
212 protein clustering has been implicated, and on the
213 modeling approaches used to approach this unique set
214 of challenges.

215 *Our Group's Focus: Spatial Aspects of Signaling* 216 *Through the Epidermal Growth Factor Receptor*

217 A member of the ErbB family of plasma membrane
218 receptors, EGFR is critically important to many bio-
219 logical processes, including embryonic development
220 and carcinogenesis.^{10,39,89} Upon binding any one of
221 several ligands, including EGF, the ErbB receptors
222 homo- or hetero-dimerize. Dimerization is followed by
223 transphosphorylation of tyrosine residues in receptor
224 cytoplasmic tails, which enables recruitment of
225 cytosolic signaling proteins. The reader is referred
226 to Figs. 2 and 3 in the article by Telasco and

Radhakrishnan⁷⁴ within this same issue, for diagrams
of EGFR/ErbB1 dimerization, phosphorylation, and
adaptor protein recruitment. Subsequently, these
complexes activate many different signaling cascades,
including the Ras-MAPK pathway discussed in the
next section.

There exists considerable experimental evidence for
preexisting clusters of resting EGFR (Fig. 3) and for
dynamical changes after addition of ligand.^{1,6,39,66,73,89}
We have built simulation platforms at different levels
of complexity, in order to evaluate the impact of
EGFR clustering in the plasma membrane.

Approaches and Methodology

Our first attempt to develop a spatial model of the
EGFR pathway was a simple compartmental model
that accounted for receptor density differences
observed in the plasma membrane, with some regions
having high-receptor density and others displaying
low-receptor density.⁵² The focus of this study was to
explore whether the added computational complexity
associated with spatial modeling was justified. Our
initial goal was to determine if the non-uniform
receptor distribution in the cell membrane could
account for the experimentally observed, concave-up
Scatchard plot for binding of EGF ligand to its
receptor. We simply optimized the distribution of
receptors into high- and low-density regions, and were
able to determine the parameter space that allowed for
a concave-up Scatchard plot. This first attempt at
compartmentalized spatial modeling showed that

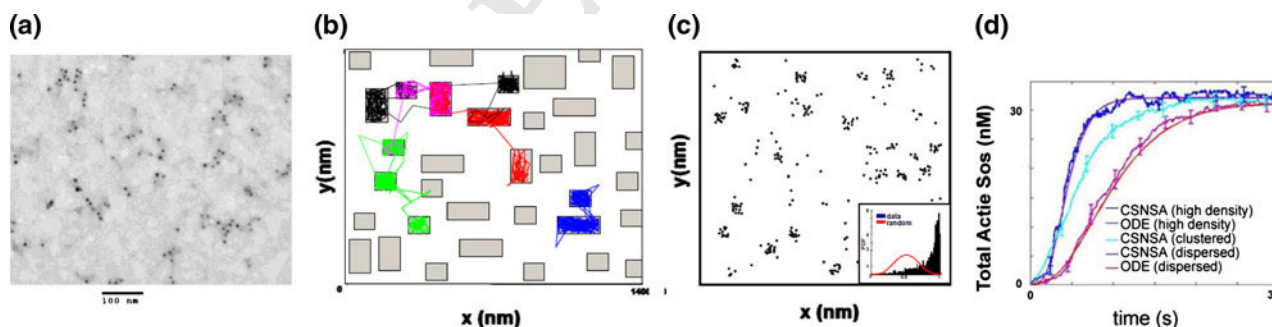


FIGURE 3. Experimental results and mathematical model predictions of EGFR clustering. (a) Experimental evidence for EGFR clustering in absence of ligand. Electron micrograph of gold particle-labeled EGF receptors in resting A341 cells (~2 million EGFR/cell), reveals a non-random distribution and provides evidence for receptor co-confinement. (b) Spatial domain used in lattice-free Monte Carlo simulation³⁵ The spatial domain simulated by the off-lattice Monte Carlo procedure was a square of area $2 \mu\text{m}^2$, representative of a small region in the plasma membrane. This region was modified to include many islands or preferred domains (the green rectangles within the membrane patch), to simulate the receptor-trapping microdomains seen in (a). Movement of receptors into and out of the simulated microdomains over a time period of 30 s is indicated by the thin colored tracings. Receptor trapping in the microdomains was reproduced mathematically by stipulating that receptors had a greater probability of entering these regions than of leaving them. (c) Simulation predictions of receptor clustering in absence of ligand. The predicted particle positions after 30 s of simulation time are indicated by the black dots. The Hopkins statistical test (inset) was used to test the randomness of receptor distribution. The right shift of the distribution (compared to the random or normal distribution shown in red) towards unity confirms the clustered nature of the receptors. The predicted receptor distribution compares well with the experimental observation in (a). (d) Simulations using a coupled spatial/nonspatial stochastic algorithm (CSNSA) support the conclusion that EGFR clustering promotes activation of the adaptor SOS. ODE models confirm this conclusion, using a fast diffusion coefficient to override contributions from membrane spatial organization (from Hsieh *et al.*³⁵ and Costa *et al.*¹⁷).

257 accounting for the spatial organization of receptors
 258 was highly valuable, and should be pursued, to enable
 259 both qualitative and quantitative understanding of cell
 260 signaling involving (at least) the EGFR.

261 This study convinced us of the utility of spatial
 262 modeling of membrane-bound receptors and of its
 263 importance in understanding cell signaling. We have
 264 now accumulated extensive experience in developing
 265 spatially realistic simulations of the cell membrane
 266 and also addressed the initiation of signal-
 267 ing.^{13,17,18,35,36,51-54} Next, we summarize our develop-
 268 ment of lattice-based and lattice-free (or off-lattice)
 269 methods, as well as our use of hybrid approaches.

270 *Lattice-Based Monte Carlo (MC) Approaches*

271 In lattice-based models, molecules are located at
 272 discrete grid points in the spatial domain and diffusion
 273 is restricted to movement to an unoccupied neighbor-
 274 ing point. Lattice-based MC simulations have become
 275 very popular in the physics, chemistry, materials,
 276 and engineering communities, as they provide
 277 spatio-temporal information at significantly reduced
 278 computational cost, compared to off-lattice simula-
 279 tions.^{5,14,16,28,90} The MC method is a coarse graining of
 280 molecular dynamics (MD) simulations,⁵ because MD
 281 is impractical for rare event dynamics, such as hopping
 282 between deep minima of a potential energy surface.
 283 The MC method stochastically solves an underlying
 284 master equation using pseudo-random numbers, by
 285 constructing the probability with which the various
 286 states of the system have to be weighted according to a
 287 Markov process. MC simulations can provide contin-
 288 uous time information. Gillespie^{26,27} established the
 289 foundations of time-dependency for chemical reactions
 290 in a spatially homogeneous system. His approach is
 291 easily applicable to arbitrary complex computational
 292 systems, and is often referred to as the *kinetic or*
 293 *dynamic MC* method. Despite important algorithmic
 294 implementations (e.g., dependency graphs,²⁵ lists of
 295 neighbors, binary-tree search, etc.), MC simulations
 296 are seriously plagued by (1) the presence of fast reac-
 297 tions that occur in large biochemical networks
 298 seen in biology and (2) the execution of one event at
 299 a time.

300 Our Spatial Kinetic Monte Carlo (SKMC)
 301 method^{52,53} utilizes a modified null-event, lattice-based
 302 MC algorithm, as in Mayawala *et al.*^{18,54} The spatial
 303 domain, represented by a small region of the plasma
 304 membrane, is a two-dimensional square lattice of side
 305 ℓ , divided into a large number of much smaller square
 306 bins of side a ($\ll \ell$). The SKMC algorithm consists of
 307 first randomly selecting an occupied lattice site, and
 308 then choosing either a successful (reaction or diffusion)
 309 or unsuccessful (null) event, based on calculated

probabilities. If a successful event is chosen, it is executed. The transition rate $\Gamma_{i \rightarrow j}^d$, for diffusion of species from any site i (i.e., lattice point i) to a nearest-neighbor site j is defined as

$$\Gamma_{i \rightarrow j}^d = \frac{1}{4} \Gamma^D \sigma_i (1 - \sigma_j), \quad j \in B_i,$$

where $\Gamma^d = 4D/a^2$ and D is the diffusion coefficient of the species located at site i . The term B_i denotes the set of four possible nearest-neighbor sites to which diffusion can occur in two dimensions from site i . Because species are allowed to diffuse only to an unoccupied site, we define an occupancy function σ_j for each of the four nearest-neighbor sites, in order to simplify the procedure for computing the transition rate for diffusion. For any site k ($= i$ or j), σ_k is set equal to 1 if the site is occupied, or to 0 if the site is unoccupied. The transition rate for a chemical reaction at site i , Γ_i^r , depends on the reaction type and is directly related to the standard reaction rate.

The probability p_i^x of an event x ($= r$ reaction or d diffusion) at site i is computed by using the relation

$$p_i^x = \Gamma_i^x / \Gamma_{\max}$$

where Γ_{\max} is a normalization constant, defined as

$$\Gamma_{\max} = 4 \left(\frac{\Gamma^d}{4} + \max \left(\sum_{\text{all forward reaction events}} \Gamma^r \right) \right) + \max \left(\sum_{\text{all backward reaction events}} \Gamma^r \right),$$

where the multiplicative factor of 4 accounts for events occurring in the four directions of the two-dimensional square lattice. Finally, the time step Δt used to advance the simulation time is computed as $\Delta t = 1/\Gamma_{\max}$.

Rule-Based, Non-lattice Simulator

Our non-lattice, stochastic simulator is an alternative approach.^{35,36} In the lattice-free method, particles are not confined to discrete points in space but are randomly repositioned upon undergoing a diffusion event. Receptors and other proteins in the 2D membrane and 3D cytosolic space are represented by sphere-like particles with radii determined from experimental data and their coarse-grained molecular models. At each time step, species diffusion is simulated as Brownian motion (Fig. 3). In addition, species have the potential to react with spatially nearby species. This simulator was designed for flexible model development and deployment by a modularized and rule-based approach. It tracks the individual reactions of multistate molecules and accommodates complex situations.

354 *Hybrid Approaches*

355 We continue to improve our basic SKMC algo-
 356 rithm, leading to increased efficiency and speed of the
 357 simulations. One significant advance was the coupling
 358 of our lattice-based SKMC simulations on the cell
 359 membrane to well-mixed stochastic simulations within
 360 the cytosol.¹⁸ In Costa *et al.*,¹⁸ we describe the devel-
 361 opment of an algorithm that couples a spatial sto-
 362 chastic model of membrane receptors with a nonspatial
 363 stochastic model of cytosolic reactions. Our novel
 364 hybrid algorithm provided a computationally efficient
 365 method to evaluate the effects of spatial heterogeneity
 366 on the coupling of receptors to cytosolic signaling
 367 partners. Results obtained using a compartmental
 368 ODE method compared well with those generated with
 369 our hybrid model. Thus, for sufficiently high receptor
 370 copy number, the far simpler ODE model may be
 371 adequate. However, for spatially inhomogenous sys-
 372 tems where the receptors numbers are low, the hybrid
 373 method was significantly superior to the ODE model.

374 *EGFR Density, Through Clustering or Overexpression,*
375 *Influences Signaling Output*

376 We have applied these methods to study the early
 377 molecular mechanisms involved in EGFR signaling.
 378 For example, we applied the lattice-based spatial sto-
 379 chastic model of the plasma membrane to examine the
 380 influence of cytoskeletal corral openings on EGFR
 381 clustering.¹⁷ Clustering was shown to depend on both
 382 receptor concentration and picket fence density. For
 383 high picket fence densities, clustering increased with
 384 increasing receptor concentration in the range exam-
 385 ined. Conversely, low receptor concentrations com-
 386 bined with small corral sizes inhibited clustering; at
 387 normal to high receptor concentration, maximal clus-
 388 tering occurred at an intermediate corral size
 389 (~100 nm). These results indicate that both the number
 390 of clusters and the average cluster size are likely to be
 391 complex functions of receptor density and microdo-
 392 main size. It follows that compartmentalization of the
 393 plasma membrane could either *inhibit* or *enhance* sig-
 394 naling, concepts that require further exploration.

395 The non-lattice, rules-based simulator allowed us to
 396 explore the effect of EGFR overexpression and its
 397 relation to carcinogenesis.³⁵ We postulated that
 398 increased receptor density in membrane microdomains
 399 or protein islands might lead to more frequent inter-
 400 actions between non-ligand bound receptors and,
 401 further, that large numbers of these short-lived inter-
 402 actions might explain EGFR signaling known to occur
 403 even in the absence of ligand.⁶ One important aspect
 404 was consideration of EGFR extracellular domain
 405 conformation, based upon structural studies showing
 406 that the resting EGFR is predominantly in a “closed”

conformation. Binding of ligand is proposed to stabi- 407
 lize the extended conformation and expose the dimer- 408
 ization arm. In our simulations, we assumed that the 409
 resting EGFR “fluxes” between the open and closed 410
 states, but spends 99% of its time in the closed state. 411
 This property translates to a low probability that two 412
 diffusing monomers will collide under conditions 413
 where both expose their dimerization arms and are 414
 therefore competent to form a complex. The 2D sim- 415
 ulation space included membrane microdomains that 416
 transiently trapped receptors (as in Fig. 3), setting up 417
 clusters undergoing dynamic exchange. Remarkably, 418
 at levels of receptors typical of most normal cells, 419
 co-confinement in membrane microdomains lowered 420
 the threshold for ligand-independent receptor dimer- 421
 ization but resulted in very modest signaling output. 422
 When the simulation space was populated with densi- 423
 ties typically seen in tumors with EGFR gene ampli- 424
 fication, which can express millions of EGFR per cell, 425
 the percent of activated receptors could exceed 10% 426
 with our parameter values. Clustering had little effect 427
 in these cases, since the overall density on the mem- 428
 brane was already very high. 429

430 We have used both lattice and non-lattice models to
 431 consider how spatial aspects might affect the recruit-
 432 ment of signaling molecules to the phosphorylated
 433 EGFR tail.^{18,36} In Hsieh *et al.*,³⁶ we also considered
 434 the combinatorial complexities associated with the fact
 435 that EGFR has multiple phosphorylation sites and,
 436 further, the fact that each phosphotyrosine site is
 437 capable of binding multiple partners. We used coarse-
 438 grained molecular docking simulations to show that
 439 steric hinderance can impose important constraints on
 440 the composition of adaptor proteins capable of dock-
 441 ing simultaneously on the EGFR tail. Modeling pre-
 442 dictions in Hsieh *et al.*³⁶ were quantitatively consistent
 443 with experimental data for the kinetics of both EGFR
 444 phosphorylation and recruitment of adaptor proteins.
 445 *Importantly, both papers provide mathematical support*
 446 *for the conclusion that clustering of receptors can*
 447 *amplify signaling by promoting sequential binding of*
 448 *adaptor proteins.* These results provide confidence in
 449 our models, and have led to ongoing studies of other
 450 growth factor receptors that initiate signaling through
 451 dimerization, particularly VEGFR, as well the hete-
 452 rodimerizing members of the ErbB family. This field
 453 continues to advance, as demonstrated by the hybrid
 454 approaches of Radhakrishnan and colleagues⁷⁴ that
 455 consider ErbB structural and diffusion properties using
 456 increasingly complex models. Additional aspects of cell
 457 surface topography, such as the induction of mem-
 458 brane curvature by endocytic adaptor proteins, are
 459 new concepts that will provide important insight into
 460 the control of signal transduction through the bio-
 461 physical principals of membranes.

462 *Work by Others: The Case of Signaling*
463 *via Ras/MAPK Pathways*

464 The Ras superfamily consists of over 100 small
465 GTP-binding proteins (or GTPases), which respond to
466 various extracellular stimuli to regulate important
467 signal transduction pathways.^{81,85} These proteins,
468 which have low intrinsic GTPase activity, “switch”
469 between active GTP-bound and inactive GDP-bound
470 conformations. The processes mediated by GTPases
471 include cell division, differentiation, apoptosis, and
472 migration, cytoskeletal reorganization, and intracellu-
473 lar protein trafficking.⁷⁵ Abnormalities in these path-
474 ways are seen in various pathologies, including obesity,
475 diabetes, inflammatory diseases, cardiovascular dis-
476 ease, neurological disease, and cancer.^{15,75,81} Therefore
477 the pharmacological targeting of GTPases and/or their
478 signaling pathways is an active field.⁸¹

479 The Ras/Raf/MEK/ERK mitogen-activated protein
480 kinase (MAPK) pathway has been investigated exten-
481 sively, both in the clinic and the laboratory, and by
482 mathematical modeling.^{7,22,23,32,34,40,41,57,61,68,69,76,77,86}
483 Activation of a number of receptors, including EGFR,
484 leads to guanine nucleotide exchange (dissociation of
485 GDP, gain of GTP) by membrane-tethered Ras,
486 thereby activating it. The activated Ras in turn acti-
487 vates Raf (Ras-associated factor), the first kinase in the
488 cascade. Subsequently, Raf activates MEK (MAPK/
489 extracellular signal-regulated kinase), which then acti-
490 vates ERK (extracellular signal-regulated kinase). The
491 translocation of phosphorylated ERK to the nucleus
492 and activation of transcription factors mediates many
493 cellular activities.

494 Numerous mathematical models have been devel-
495 oped to study this pathway.^{7,23,32,34,40,41,57,61,68,69,76,77,86}
496 Much of this work uses compartmental models and
497 ODEs to follow the temporal evolution of activated
498 ERK, and does not consider clustering in the plasma
499 membrane. However, Tian *et al.*^{76,77} have mathemati-
500 cally evaluated various spatial aspects of Ras signaling,
501 including clustering in the plasma membrane. This
502 group utilized a hybrid approach to simulate reactions in
503 the cell membrane and those in the cytosol, enabling
504 them to separate the contribution of the plasma mem-
505 brane structure to the signal. They combined the well-
506 mixed stochastic model of Gillespie^{26,27} to simulate
507 reactions in the membrane with an ODE model for the
508 cytosolic reactions. They assumed that the number of
509 RasGTP clusters was proportional to the EGF con-
510 centration, and these clusters served as platforms for
511 recruiting Raf to the plasma membrane for activation.
512 The lifetime of RasGTP clusters was assumed to be
513 normally distributed over a measured value. Plasma
514 membrane reactions, in addition to binding and acti-
515 vation of Raf by RasGTP clusters, included recruitment

by activated Raf of the KSR–MEK–ERK complex from 516
the cytosol and activation of MEK by activated Raf and 517
of ERK (MAPK) by activated MEK. KSR (kinase 518
suppressor of Ras) is a scaffold protein that facilitates 519
MAPK activation by providing binding sites for 520
assembly of the signaling complex. The recruitment of 521
both Raf and the KSR–MEK–ERK complex was 522
modeled as occurring through random collisions with 523
the plasma membrane. With dissolution of a nanoclus- 524
ter, all recruited proteins diffused back to the cytosol, 525
where the activated MEK and ERK continued their 526
roles. Using this model in conjunction with biological 527
experiments, Tian *et al.*⁷⁶ concluded that RasGTP 528
clustering is essential for signal transduction. Moreover, 529
the RasGTP clusters operate as sensitive switches in that 530
they produce approximately the same levels of normal- 531
ized activated ERK over a wide range of ligand con- 532
centration. One possible explanation for this behavior is 533
the establishment of locally high concentrations of 534
recruited proteins and thus the spatial restriction of 535
active ERK production to RasGTP nanoclusters, whose 536
generation and lifetime are themselves strictly regu- 537
lated.⁷⁶ Tian *et al.*⁷⁶ also concluded that the produc- 538
tion of RasGTP nanoclusters in direct proportion to 539
ligand concentration can ensure high fidelity of signal 540
transduction. 541

542 Subsequently, Tian *et al.*⁷⁷ incorporated models for 542
following the temporal evolution of RasGTP clusters 543
in the cell membrane. In particular, they studied K-Ras 544
clustering and how it is influenced by the protein 545
Galectin-3 (Gal3). Previous experimental work had 546
shown that Gal3 is a scaffolding protein recruited to 547
the plasma membrane, where it is necessary for the 548
formation of Ras nanoclusters.⁷⁰ Their mathematical 549
model⁷⁷ considered the two species, membrane-bound 550
RasGTP and Gal3, initially in the cytosol. Once Gal3 551
is recruited by RasGTP, the RasGTP–Gal3 complexes 552
are assumed to diffuse randomly in the plasma mem- 553
brane and react with one another to form complexes of 554
various sizes. 555

556 To simplify the calculation procedure, Tian *et al.*⁷⁷ 556
allowed for a maximum cluster size of ten. The various 557
combinations of possible complexes resulted in a total 558
of 27 species and 136 reactions in the plasma mem- 559
brane. In agreement with our earlier observation, they 560
concluded that spatial stochastic modeling of such a 561
large system poses a considerable computational bur- 562
den. Therefore they developed an ODE system to fol- 563
low the temporal evolution of complexes of size 1–10, 564
using a spatial stochastic model to only deduce colli- 565
sion rates among the complexes.^{35,36} This determi- 566
nistic system was solved with a Runge–Kutta method 567
suitable for stiff ODEs.⁶⁰ The collision rates were 568
obtained by initially placing RasGTP randomly in a 569

570 square-shaped representation of the plasma mem-
 571 brane. Recruitment of Gal3 produces the Ras–Gal
 572 complex. These molecules were allowed to diffuse
 573 randomly, and a collision was said to occur when the
 574 distance between two molecules was less than the sum
 575 of their radii. The collisions produced various combi-
 576 nations of Ras–Gal complexes. When a nanocluster,
 577 defined as a cluster consisting of five or more RasGTP
 578 molecules, formed it was assumed to become immobile
 579 in the plasma membrane. During the calculation pro-
 580 cedure the total numbers of collisions giving rise to all
 581 cluster types were tracked. At the end of the compu-
 582 tational time period, the collision rate constants were
 583 computed from the total numbers of collisions. Kinetic
 584 rate constants for the ODE model were then derived
 585 from the collision rate constants, by using a genetic
 586 algorithm in conjunction with experimental data. The
 587 validity of this deterministic ODE model was checked
 588 with results generated with a stochastic simulation
 589 algorithm.²⁶ Presumably due to the large numbers of
 590 proteins, the stochastic simulations predicted only
 591 small fluctuations. This observation supports use of
 592 deterministic models when the protein copy number is
 593 high, in agreement with our observations.

594 Using this modeling approach, Tian *et al.*⁷⁷ studied
 595 clustering of K-Ras-GTP in the plasma membrane
 596 arising from interactions with Gal3 for various KRas
 597 and Gal3 copy numbers. The simulation time period
 598 was sufficiently long for the system to equilibrate. The
 599 time to equilibrate was approximately two minutes, an
 600 important result because it is in good agreement with
 601 the time period required for RasGTP loading in
 602 response to stimulation.⁷⁶ Their results also success-
 603 fully reproduced the experimental results of Plowman
 604 *et al.*⁵⁹ that approximately 42% of the RasGTP were in
 605 clusters and the average cluster size was approximately
 606 7. Tian *et al.*⁷⁷ also generated the equilibrium nano-
 607 cluster number vs. size histogram. Their results showed
 608 that nanoclusters with two to four molecules
 609 accounted for only 2.1% of the RasGTP, whereas a
 610 cluster size of 5 was the most prevalent. Nanoclusters
 611 larger than 5 in size were progressively smaller in
 612 number, approximately inversely proportional to the
 613 size. The authors speculate that one possible reason for
 614 the lowered incorporation of RasGTPGal3 complexes
 615 into clusters of size 5 or larger is the remodeling of the
 616 lipid environment of the cluster by the stable pentamer.
 617 Their results also suggest that cluster formation is only
 618 weakly dependent on RasGTP concentration, and is
 619 determined by the Gal3 cytosolic concentration. Tian
 620 *et al.*⁷⁷ concluded that on the basis of their simulations
 621 neglecting the formation of clusters with more than 10
 622 RasGTP molecules is reasonable. Notably, this work
 623 illustrates the difficulty of spatial modeling of systems
 624 with large reaction networks.

Work by Others: G-Protein Coupled Receptors

625
 626 The GPCRs constitute the largest family of trans-
 627 membrane receptors, consisting of five subfamilies.^{2,65}
 628 These proteins, whose structure and function were
 629 reviewed recently by Rosenbaum *et al.*,⁶⁵ are charac-
 630 terized by seven transmembrane spanning α -helical
 631 segments.^{2,24} They regulate many physiological func-
 632 tions such as vision, gustation, and olfaction.^{65,82}
 633 Neurotransmitters, hormones, and environmental
 634 stimuli activate these pathways. GPCRs are also
 635 implicated in many human diseases, such as inflam-
 636 mation, retinitis pigmentosa, nephrogenic diabetes
 637 insipidus and Kaposi's sarcoma.^{24,38,82,83} At present,
 638 most pharmaceutical drugs used by humans target
 639 GPCRs by serving as agonists or antagonists.^{21,82}

640 Many aspects of GPCR signaling are well estab-
 641 lished. In the classical view, binding of ligand to a
 642 GPCR induces a conformational change in the recep-
 643 tor. The activated receptor initiates guanine nucleotide
 644 exchange (GDP \rightarrow GTP) in its principal signaling
 645 partner, a heterotrimeric ($\alpha\beta\gamma$) G-protein complex.
 646 Like ras, heterotrimeric G proteins are tethered to the
 647 cytosolic leaflet of the plasma membrane through
 648 covalently attached lipids, and assume an active state
 649 once bound to GTP. An additional step is required for
 650 heterotrimeric G proteins: the separation of the GTP-
 651 bound G α subunit from the G $\beta\gamma$ subunit, which dif-
 652 fuses into the cytosol. The subsequent activation of
 653 downstream effector proteins results in various distinct
 654 biological reactions.

655 Recent work has focused on new aspects of GPCR
 656 signaling, such as the evidence that at least some
 657 GPCRs can form homo- or hetero-dimers.^{8,24,83} These
 658 dimers can interact further to form oligomers.²¹
 659 Although believed essential for signaling to occur, the
 660 dimerization mechanism is well characterized for only
 661 a few GPCRs.⁴⁴ Due to the importance of GPCR
 662 signaling in healthy and diseased states, GPCR inter-
 663 actions, along with membrane organization, and their
 664 impact on signaling must be well characterized.
 665 Mathematical modeling is therefore being used
 666 increasingly to help unravel the intricacies of this
 667 pathway. A useful review of mathematical models that
 668 have been developed to study GPCR signaling is given
 669 by Linderman.⁴⁸

670 Brinkerhoff *et al.*⁸ used triangular lattice-based MC
 671 models to simulate receptor dimerization and activa-
 672 tion in a two-dimensional plane, examining how
 673 dimerization creates clusters of receptors. Their model
 674 demonstrates the applicability of MC methods to sys-
 675 tems with discrete reactions that are diffusion limited.⁸
 676 Randomly selected particles undergo either one of
 677 two possibilities at each time step: displacement in a
 678 random direction by a distance governed by the

679 diffusion coefficient or a chemical reaction. Reaction
680 possibilities considered were receptor dimerization,
681 binding of ligand by receptor, receptor activation of G
682 protein and receptor phosphorylation. This group's
683 simulations suggest that clustering arises through both
684 dimerization and cross talk between receptors as they
685 approach one another closely and are able to share an
686 effector. They also concluded that the resulting clus-
687 tering enhances signaling.

688 Fallahi-Sichani and Linderman²¹ investigated lipid
689 raft impact on GPCR signaling with a combination of
690 MC (stochastic) and deterministic models. A lattice-
691 based, kinetic MC model was used to establish the
692 effects of low-diffusivity rafts on receptor dimerization
693 and cluster dynamics. The stochasticity of the model
694 allowed for receptor distributions to be examined,
695 leading to parameter estimations for exploring the
696 effects on downstream signaling using an ODE model.
697 The fraction of plasma membrane covered by micro-
698 domains (rafts), which was varied from 2 to 30%, had
699 a significant impact on output. At 2% coverage,
700 microdomains amplified the overall response, but at
701 higher coverage the signal was attenuated. They con-
702 cluded that dimerization and lipid raft trapping
703 cooperatively control the extent and dynamics of
704 GPCR signaling.

705 Tolle and Le Novere⁷⁸ developed an off-lattice,
706 Brownian diffusion-based stochastic model, which they
707 used to determine how alpha-amino-3-hydroxyl-5-
708 methyl-4-isoxazolepropionic acid receptor (AMPA)
709 diffusion in the dendritic spine affects synaptic signaling,
710 specifically long-term potentiation (LTP).⁷⁹ LTP, an
711 increase in synaptic strength, is a well-studied form of
712 synaptic plasticity, the ability to change the strength of
713 a signal.^{67,79} Tolle and Le Novere's⁷⁹ model accounts
714 for the dendritic spine membrane, membrane recep-
715 tors, and scaffolding proteins known to bind to
716 membrane receptors. The spatial domain representing
717 the plasma membrane of the synaptic spine was mode-
718 led as a square of surface area corresponding to the
719 measured volume of the spine. This square was sepa-
720 rated into two different compartments or domains, in
721 order to account for the two physiologically different
722 portions of the plasma membrane: the post-synaptic
723 density (PSD) and the extra-synaptic membrane
724 (ESM). The PSD is a protein-rich region where
725 AMPARs are concentrated,^{67,79} while the rest of the
726 membrane is classified as the ESM.⁷⁹ The transmem-
727 brane receptor movement within the ESM was mode-
728 led with Brownian-type diffusion, while confined
729 motion was used to model the restricted diffusion
730 within the PSD. Simulation results indicate that ran-
731 domly placed receptors quickly localize to the PSD,
732 which Tolle and Le Novere⁷⁹ suggest explains the
733 quick onset of LTP.

CONCLUDING REMARKS

This review specifically considers the mathematical modeling of protein clustering on the plasma membrane and the evidence that signal transduction can be enhanced by locally high concentrations of proteins that increase the probability of protein-protein interactions. This feature is especially important when the numbers of particles are small. When proteins are overexpressed, as in EGFR amplification in certain cancers, clustering may not be as significant.³⁵ The role of membrane microdomains in signaling may be quite complex, since both inhibitory and stimulatory effects have been observed experimentally and theoretically.^{3,17,55,58}

Mathematical modeling, in conjunction with biological experiments, is providing new insights into the mechanisms that govern protein clustering in membranes and the resulting impact on signaling. Increasing experimental detail is being matched by increasingly complex models that account for previously ignored biological subtleties.^{12,19,30,45,62,64,80} An important goal is to predict the functional responses of whole cells and cell-tissue systems, based upon integration of spatial and temporally encoded signals from surface receptors. Achieving this goal will necessitate the development of efficient and accurate multi-scale simulation capabilities. A daunting challenge to mathematical modeling of cell signaling continues to be the scaling up of computationally intense methods developed for studying molecular behavior to enable predictive modeling at progressively more complex levels, from the cellular to the systemic.

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REFERENCES

- Abulrob, A., *et al.* Nanoscale imaging of epidermal growth factor receptor clustering: effects of inhibitors. *J. Biol. Chem.* 285(5):3145–3156, 2010.
- Alberts, B. *Molecular Biology of the Cell* (5th ed.). New York: Garland Science, 2008. 1 v. (various pagings).
- Allen, J. A., R. A. Halverson-Tamboli, and M. M. Rasenick. Lipid raft microdomains and neurotransmitter signalling. *Nat. Rev. Neurosci.* 8(2):128–140, 2007.
- Andrews, S. S., and D. Bray. Stochastic simulation of chemical reactions with spatial resolution and single molecule detail. *Phys. Biol.* 1(3–4):137–151, 2004.

- 785 ⁵Auerbach, S. M. Theory and simulation of jump dynamics, 849
786 diffusion and phase equilibrium in nanopores. *Int. Rev.* 850
787 *Phys. Chem.* 19(2):155–198, 2000. 851
788 ⁶Bader, A. N., *et al.* Homo-FRET imaging enables quan- 852
789 tification of protein cluster sizes with subcellular resolu- 853
790 tion. *Biophys. J.* 97(9):2613–2622, 2009. 854
791 ⁷Brightman, F. A., and D. A. Fell. Differential feedback 855
792 regulation of the MAPK cascade underlies the quantitative 856
793 differences in EGF and NGF signalling in PC12 cells. 857
794 *FEBS Lett.* 482(3):169–174, 2000. 858
795 ⁸Brinkerhoff, C. J., P. J. Woolf, and J. J. Linderman. Monte 859
796 Carlo simulations of receptor dynamics: insights into cell 860
797 signaling. *J. Mol. Histol.* 35(7):667–677, 2004. 861
798 ⁹Brown, G. C., and B. N. Kholodenko. Spatial gradients of 862
799 cellular phospho-proteins. *FEBS Lett.* 457(3):452–454, 863
800 1999. 864
801 ¹⁰Bublil, E. M., and Y. Yarden. The EGF receptor family: 865
802 spearheading a merger of signaling and therapeutics. *Curr.* 866
803 *Opin. Cell Biol.* 19(2):124–134, 2007. 867
804 ¹¹Burrage, K., *et al.* Modelling and simulation techniques for 868
805 membrane biology. *Brief. Bioinform.* 8(4):234–244, 2007. 869
806 ¹²Chakraborty, A. K., M. L. Dustin, and A. S. Shaw. In 870
807 silico models for cellular and molecular immunology: suc- 871
808 cesses, promises and challenges. *Nat. Immunol.* 4(10):933– 872
809 936, 2003. 873
810 ¹³Chatterjee, A., *et al.* Time accelerated Monte Carlo simu- 874
811 lations of biological networks using the binomial tau-leap 875
812 method. *Bioinformatics* 21(9):2136–2137, 2005. 876
813 ¹⁴Chuan Kang, H., and W. Weinberg. Modeling the kinetics 877
814 of heterogeneous catalysis. *Chem. Rev.* 95:667–676, 1995. 878
815 ¹⁵Colicelli, J. Human RAS superfamily proteins and related 879
816 GTPases. *Sci. STKE* 2004(250):RE13, 2004. 880
817 ¹⁶Coppens, M. O., A. T. Bell, and A. K. Chakraborty. 881
818 Dynamic Monte-Carlo and mean-field study of the effect of 882
819 strong adsorption sites on self-diffusion in zeolites. *Chem.* 883
820 *Eng. Sci.* 54:3455–3463, 1999. 884
821 ¹⁷Costa, M. N., K. Radhakrishnan, and J. S. Edwards. 885
822 Monte Carlo simulations of plasma membrane corral-in- 886
823 duced EGFR clustering. *J. Biotechnol.* 151(3):261–270, 887
824 2009. 888
825 ¹⁸Costa, M. N., *et al.* Coupled stochastic spatial and non- 889
826 spatial simulations of ErbB1 signaling pathways demon- 890
827 strate the importance of spatial organization in signal 891
828 transduction. *PLoS ONE* 4(7):e6316, 2009. 892
829 ¹⁹Erban, R., and S. J. Chapman. Stochastic modelling of 893
830 reaction–diffusion processes: algorithms for bimolecular 894
831 reactions. *Phys. Biol.* 6(4):046001, 2009. 895
832 ²⁰Faeder, J., M. Blinov, and W. Hlavacek. Rules-based 896
833 modeling of biochemical systems with BioNetGen. *Meth-* 897
834 *ods Mol. Biol.* 500:113–168, 2009. 898
835 ²¹Fallahi-Sichani, M., and J. J. Linderman. Lipid raft-med- 899
836 iated regulation of G-protein coupled receptor signaling by 900
837 ligands which influence receptor dimerization: a computa- 901
838 tional study. *PLoS ONE* 4(8):e6604, 2009. 902
839 ²²Friday, B. B., and A. A. Adjei. Advances in targeting the 903
840 Ras/Raf/MEK/Erk mitogen-activated protein kinase cas- 904
841 cade with MEK inhibitors for cancer therapy. *Clin. Cancer* 905
842 *Res.* 14(2):342–346, 2008. 906
843 ²³Fujioka, A., *et al.* Dynamics of the Ras/ERK MAPK 907
844 cascade as monitored by fluorescent probes. *J. Biol. Chem.* 908
845 281(13):8917–8926, 2006. 909
846 ²⁴Fuxe, K., and T. Kenakin. The changing world of G pro- 910
847 tein-coupled receptors. *J. Recept. Signal Transduct. Res.* 911
848 30(5):271, 2010. 912
- ²⁵Gibson, M. A., and J. Bruck. Efficient exact stochastic 849
simulation of chemical systems with many species and 850
many channels. *J. Phys. Chem.* 104:1876–1889, 2000. 851
²⁶Gillespie, D. T. Exact stochastic simulation of coupled 852
chemical reactions. *J. Phys. Chem.* 81(25):2340–2361, 1977. 853
²⁷Gillespie, D. T. Stochastic simulation of chemical kinetics. 854
Annu. Rev. Phys. Chem. 58:35–55, 2007. 855
²⁸Gilmer, G. Computer models of crystal growth. *Science* 856
208:355–363, 1980. 857
²⁹Govindan, R. A review of epidermal growth factor recep- 858
tor/HER2 inhibitors in the treatment of patients with non- 859
small-cell lung cancer. *Clin. Lung Cancer* 11(1):8–12, 2010. 860
³⁰Grima, R., and S. Schnell. Modelling reaction kinetics 861
inside cells. *Essays Biochem.* 45:41–56, 2008. 862
³¹Hartman, N. C., and J. T. Groves. Signaling clusters in the 863
cell membrane. *Curr. Opin. Cell Biol.* 23(4):370–376, 2011. 864
³²Hatakeyama, M., *et al.* A computational model on the 865
modulation of mitogen-activated protein kinase (MAPK) 866
and Akt pathways in heregulin-induced ErbB signalling. 867
Biochem. J. 373(Pt 2):451–463, 2003. 868
³³Hlavacek, W., *et al.* Rules for modeling signal-transduction 869
systems. *Sci. STKE* 2006:re6, 2006. 870
³⁴Hornberg, J. J., *et al.* Control of MAPK signalling: from 871
complexity to what really matters. *Oncogene* 24(36):5533– 872
5542, 2005. 873
³⁵Hsieh, M. Y., *et al.* Stochastic simulations of ErbB homo 874
and heterodimerisation: potential impacts of receptor 875
conformational state and spatial segregation. *IET Syst.* 876
Biol. 2(5):256–272, 2008. 877
³⁶Hsieh, M. Y., *et al.* Spatio-temporal modeling of signaling 878
protein recruitment to EGFR. *BMC Syst. Biol.* 4:57, 2010. 879
³⁷Hynes, N. E., and G. MacDonald. ErbB receptors and 880
signaling pathways in cancer. *Curr. Opin. Cell Biol.* 881
21(2):177–184, 2009. 882
³⁸Insel, P. A., *et al.* Impact of GPCRs in clinical medicine: 883
monogenic diseases, genetic variants and drug targets. 884
Biochim. Biophys. Acta 1768(4):994–1005, 2007. 885
³⁹Keating, E., A. Nohe, and N. O. Petersen. Studies of dis- 886
tribution, location and dynamic properties of EGFR on the 887
cell surface measured by image correlation spectroscopy. 888
Eur. Biophys. J. 37(4):469–481, 2008. 889
⁴⁰Kholodenko, B. N., J. F. Hancock, and W. Kolch. Sig- 890
nalling ballet in space and time. *Nat. Rev. Mol. Cell Biol.* 891
11(6):414–426, 2010. 892
⁴¹Kholodenko, B. N., *et al.* Quantification of short term 893
signaling by the epidermal growth factor receptor. *J. Biol.* 894
Chem. 274(42):30169–30181, 1999. 895
⁴²Kitaura, J., *et al.* Evidence that IgE molecules mediate a 896
spectrum of effects on mast cell survival and activation via 897
aggregation of the FcepsilonRI. *Proc. Natl. Acad. Sci. USA* 898
100(22):12911–12916, 2003. 899
⁴³Kusumi, A., *et al.* Paradigm shift of the plasma membrane 900
concept from the two-dimensional continuum fluid to the 901
partitioned fluid: high-speed single-molecule tracking of 902
membrane molecules. *Annu. Rev. Biophys. Biomol. Struct.* 903
34:351–378, 2005. 904
⁴⁴Lambert, N. A. GPCR dimers fall apart. *Sci. Signal.* 905
3(115):pe12, 2010. 906
⁴⁵Li, H., *et al.* Algorithms and software for stochastic sim- 907
ulation of biochemical reacting systems. *Biotechnol. Prog.* 908
24(1):56–61, 2008. 909
⁴⁶Lidke, D. S., and B. S. Wilson. Caught in the act: quanti- 910
fying protein behaviour in living cells. *Trends Cell Biol.* 911
19(11):566–574, 2009. 912

- 913 47Lillemeier, B. F., *et al.* TCR and Lat are expressed on
914 separate protein islands on T cell membranes and concat-
915 enate during activation. *Nat. Immunol.* 11(1):90–96, 2010.
- 916 48Linderman, J. J. Modeling of G-protein-coupled receptor
917 signaling pathways. *J. Biol. Chem.* 284(9):5427–5431, 2009.
- 918 49Lo, H. W. Nuclear mode of the EGFR signaling network:
919 biology, prognostic value, and therapeutic implications.
920 *Discov. Med.* 10(50):44–51, 2010.
- 921 50Low-Nam, S. T., *et al.* ErbB1 dimerization is promoted by
922 domain co-confinement and stabilized by ligand binding.
923 *Nat. Struct. Mol. Biol.* 18(11):1244–1249, 2011.
- 924 51Mayawala, K., C. A. Gelmi, and J. S. Edwards. MAPK
925 cascade possesses decoupled controllability of signal
926 amplification and duration. *Biophys. J.* 87(5):L01–L02,
927 2004.
- 928 52Mayawala, K., D. G. Vlachos, and J. S. Edwards. Hetero-
929 geneities in EGF receptor density at the cell surface can
930 lead to concave up scatchard plot of EGF binding. *FEBS
931 Lett.* 579(14):3043–3047, 2005.
- 932 53Mayawala, K., D. G. Vlachos, and J. S. Edwards. Com-
933 putational modeling reveals molecular details of epidermal
934 growth factor binding. *BMC Cell Biol.* 6:41, 2005.
- 935 54Mayawala, K., D. G. Vlachos, and J. S. Edwards. Spatial
936 modeling of dimerization reaction dynamics in the plasma
937 membrane: Monte Carlo vs. continuum differential equa-
938 tions. *Biophys. Chem.* 121(3):194–208, 2006.
- 939 55Miura, Y., K. Hanada, and T. L. Jones. G(s) signaling is
940 intact after disruption of lipid rafts. *Biochemistry*
941 40(50):15418–15423, 2001.
- 942 56Nagy, P., *et al.* Lipid rafts and the local density of ErbB
943 proteins influence the biological role of homo- and hete-
944 roassociations of ErbB2. *J. Cell Sci.* 115(Pt 22):4251–4262,
945 2002.
- 946 57Orton, R. J., *et al.* Computational modelling of the
947 receptor-tyrosine-kinase-activated MAPK pathway. *Bio-
948 chem. J.* 392(Pt 2):249–261, 2005.
- 949 58Pike, L. J. Lipid rafts: bringing order to chaos. *J. Lipid Res.*
950 44(4):655–667, 2003.
- 951 59Plowman, S. J., *et al.* H-ras, K-ras, and inner plasma
952 membrane raft proteins operate in nanoclusters with dif-
953 ferential dependence on the actin cytoskeleton. *Proc. Natl.
954 Acad. Sci. USA* 102(43):15500–15505, 2005.
- 955 60Radhakrishnan, K. Combustion kinetics and sensitivity
956 analysis. In: Numerical Approaches to Combustion Mod-
957 eling, edited by E. S. Oran, and J. P. Boris. Washington,
958 DC: AIAA, 1991, pp. 83–128.
- 959 61Radhakrishnan, K., *et al.* Sensitivity analysis predicts that
960 the ERK–pMEK interaction regulates ERK nuclear
961 translocation. *IET Syst. Biol.* 3(5):329–341, 2009.
- 962 62Radhakrishnan, K., *et al.* Quantitative understanding of
963 cell signaling: the importance of membrane organization.
964 *Curr. Opin. Biotechnol.* 21(5):677–682, 2010.
- 965 63Reddy, A. S., S. Chilukuri, and S. Raychaudhuri. The
966 network of receptors characterize B cell receptor micro-
967 and macroclustering in a Monte Carlo model. *J. Phys.
968 Chem. B* 114(1):487–494, 2010.
- 969 64Resat, H., L. Petzold, and M. F. Pettigrew. Kinetic mod-
970 eling of biological systems. *Methods Mol. Biol.* 541:311–
971 335, 2009.
- 972 65Rosenbaum, D. M., S. G. Rasmussen, and B. K. Kobilka.
973 The structure and function of G-protein-coupled receptors.
974 *Nature* 459(7245):356–363, 2009.
- 975 66Saffarian, S., *et al.* Oligomerization of the EGF receptor
976 investigated by live cell fluorescence intensity distribution
977 analysis. *Biophys. J.* 93(3):1021–1031, 2007.
- 978 67Santamaria, F., *et al.* Quantifying the effects of elastic
979 collisions and non-covalent binding on glutamate receptor
980 trafficking in the post-synaptic density. *PLoS Comput. Biol.*
981 6(5):e1000780, 2010.
- 982 68Sasagawa, S., *et al.* Prediction and validation of the distinct
983 dynamics of transient and sustained ERK activation. *Nat.
984 Cell Biol.* 7(4):365–373, 2005.
- 985 69Schoeberl, B., *et al.* Computational modeling of the
986 dynamics of the MAP kinase cascade activated by surface
987 and internalized EGF receptors. *Nat. Biotechnol.*
988 20(4):370–375, 2002.
- 989 70Shalom-Feuerstein, R., *et al.* K-ras nanoclustering is sub-
990 verted by overexpression of the scaffold protein galectin-3.
991 *Cancer Res.* 68(16):6608–6616, 2008.
- 992 71Slepchenko, B. M., *et al.* Quantitative cell biology with the
993 virtual cell. *Trends Cell Biol.* 13(11):570–576, 2003.
- 994 72Suzuki, K., *et al.* Rapid hop diffusion of a G-protein-
995 coupled receptor in the plasma membrane as revealed by
996 single-molecule techniques. *Biophys. J.* 88(5):3659–3680,
997 2005.
- 998 73Szabo, A., *et al.* Quantitative characterization of the large-
999 scale association of ErbB1 and ErbB2 by flow cytometric
1000 homo-FRET measurements. *Biophys. J.* 95(4):2086–2096,
1001 2008.
- 1002 74Telasco, S., and K. Radhakrishnan. Structural systems
1003 biology and multiscale signaling models. *Ann. Biomed.
1004 Eng.*, 2012, in press.
- 1005 75ten Klooster, J. P., and P. L. Hordijk. Targeting and
1006 localized signalling by small GTPases. *Biol. Cell* 99(1):1–12,
1007 2007.
- 1008 76Tian, T., *et al.* Plasma membrane nanoswitches generate
1009 high-fidelity Ras signal transduction. *Nat. Cell Biol.*
1010 9(8):905–914, 2007.
- 1011 77Tian, T., *et al.* Mathematical modeling of K-Ras nano-
1012 cluster formation on the plasma membrane. *Biophys. J.*
1013 99(2):534–543, 2010.
- 1014 78Tolle, D. P., and N. Le Novere. Meredys, a multi-com-
1015 partment reaction–diffusion simulator using multistate
1016 realistic molecular complexes. *BMC Syst. Biol.* 4:24,
1017 2010.
- 1018 79Tolle, D. P., and N. Le Novere. Brownian diffusion of
1019 AMPA receptors is sufficient to explain fast onset of LTP.
1020 *BMC Syst. Biol.* 4:25, 2010.
- 1021 80Turner, T. E., S. Schnell, and K. Burrage. Stochastic
1022 approaches for modelling in vivo reactions. *Comput. Biol.
1023 Chem.* 28(3):165–178, 2004.
- 1024 81Vigil, D., *et al.* Ras superfamily GEFs and GAPs: validated
1025 and tractable targets for cancer therapy? *Nat. Rev. Cancer*
1026 10(12):842–857, 2010.
- 1027 82Vilaradaga, J. P., *et al.* G-protein-coupled receptor
1028 heteromer dynamics. *J. Cell Sci.* 123(Pt 24):4215–4220,
1029 2010.
- 1030 83Waller, A., *et al.* Receptor binding kinetics and cellular
1031 responses of six *N*-formyl peptide agonists in human neu-
1032 trophils. *Biochemistry* 43(25):8204–8216, 2004.
- 1033 84Wells, N. P., *et al.* Time-resolved three-dimensional
1034 molecular tracking in live cells. *Nano Lett.* 10(11):4732–
1035 4737, 2010.
- 1036 85Wennerberg, K., K. L. Rossman, and C. J. Der. The Ras
1037 superfamily at a glance. *J. Cell Sci.* 118(Pt 5):843–846,
1038 2005.
- 1039 86Wiley, H. S., S. Y. Shvartsman, and D. A. Lauffenburger.
1040 Computational modeling of the EGF-receptor system: a
1041 paradigm for systems biology. *Trends Cell Biol.* 13(1):43–
1042 50, 2003.

- 1043 ⁸⁷Wilson, B. S., J. M. Oliver, and D. S. Lidke. Spatio-temporal signaling in mast cells. *Adv. Exp. Med. Biol.* 716:91–106, 2010. 1049
- 1044 1050
- 1045 1051
- 1046 ⁸⁸Wilson, B. S., *et al.* Exploring membrane domains using native membrane sheets and transmission electron microscopy. *Methods Mol. Biol.* 398:245–261, 2007. 1052
- 1047 1053
- 1048 1054
- 1055
- ⁸⁹Yang, S., *et al.* Mapping ErbB receptors on breast cancer cell membranes during signal transduction. *J. Cell Sci.* 120(Pt 16):2763–2773, 2007.
- ⁹⁰Zhdanov, V. P., and B. Kasemo. Kinetic phase transitions in simple reactions on solid surfaces. *Surf. Sci. Rep.* 20:111–189, 1994.

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